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Supplement

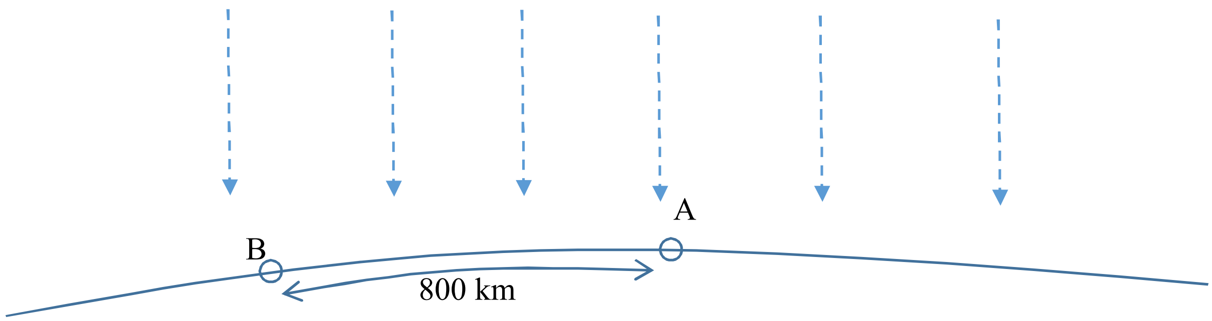
to the

Standardized Test Prep Package

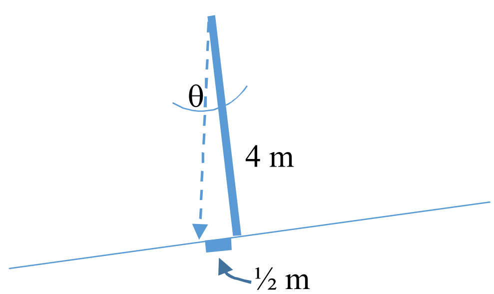
"Circumference of the earth"

Suppose you know the sun will be directly overhead Town A at 12:00 noon on June 22. At that moment, you will plan to be in Town B, 800 kilometers away. Here, using a 4 meter pole held upright, you measure a shadow that is ½ meter long. How can this information be used to estimate the circumference of the earth?

This one takes a little ingenuity in setting up in order to use arc length, trigonometry, and parallel lines to calculate. A sketch of the earth's arc shows the sun's rays coming straight down directly over Town A at 12:00:



As we zoom in on the 4 m pole in Town B, we can see the shadow formed:

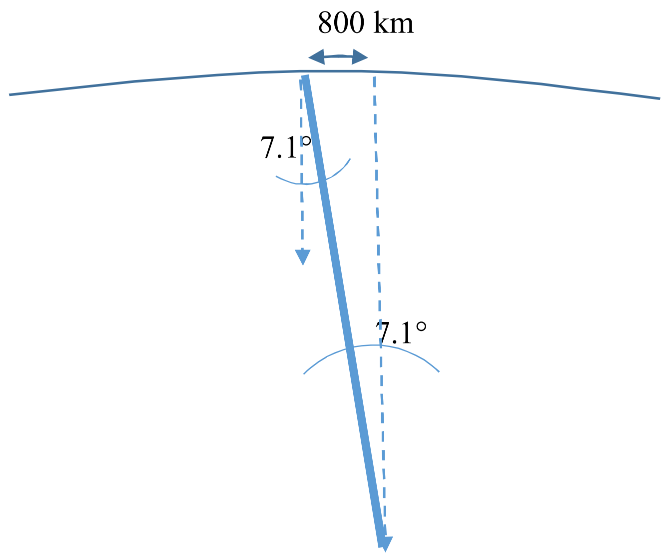


The angle θ formed at the top, between the pole and the sun's ray can be found:

$$\tan\theta = \frac{1/2}{4} = 1/8 = .125$$

$$\theta = 7.1°$$

Extending the ray at Town A and the pole in Town B to the center of the earth, we see – since the *sun's rays are parallel* - the measure of the central angle that corresponds with the arc length of 800 km is also 7.2°:



Now we can use the ratio to calculate the circumference:

$$\frac{7.1}{360} = \frac{800}{C}$$

$$C = 40{,}560 \text{ km}$$

Note: This is what Eratosthenes did around 235 B.C. to estimate the earth's circumference, where Towns A and B were Syene and Alexandria, Egypt, respectively.[1] Actual circumference ranges from 40,008 to 40,075 (due to bulge at equator), so Eratosthenes was only about 1.2% higher. Not too bad.

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[1] Hewitt, Paul G. 2010. *Conceptual Physics, 11th Edition*. Boston: Addison-Wesley.